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# THE FERMAT'S LAST THEOREM FROM THE EYE OF PHYSICIST

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Skeptics continue to believe that Pierre de Fermat was probably mistaken. It turned out that Pierre de Fermat's statement is not a figure of speech, that it should be taken literally. The mathematician did not lie at all when talking about the possibility of recording the main ideas of the proof in the fields of Diophantus arithmetic. At least six faces of the wooden cube were enough. From a philosophical standpoint, Fermat's Last theorem contains an irremediable conflict between form and content.

Keywords: *Fermat's, symmetry, number theory, topology, isotropy, homogeneity*

## Introduction

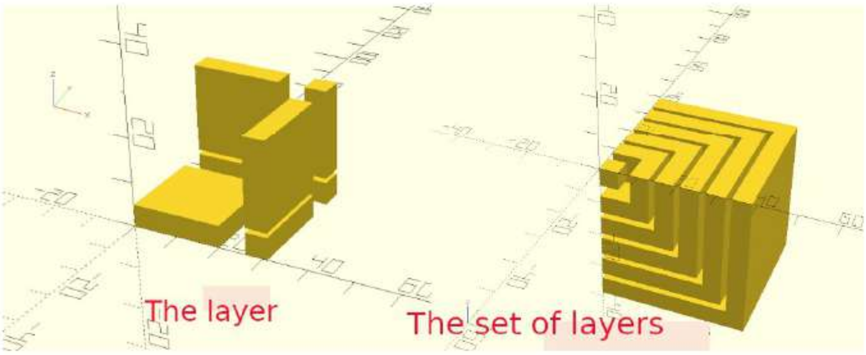
Fermat's Last Theorem was formulated by Pierre de Fermat in 1672, it states that the Diophantine equation:

$$a^n + b^n = c^n \qquad (1)$$

has no solutions in integers, except for zero values, for n > 2. The case degree of two is known in the school course under the name theorem Pythagoras. Euler in 1770 proved Theorem (1) for n=3, Dirichlet and Legendre in 1825 - for n = 5, Lame - for n = 7. In 1994 Prof. Princeton University Andrew Wiles [1] proved (1), for all n, but this proof, contains over one hundred and forty pages, understandable only to high qualified specialists in the field of number theory.

## But there is also a brief proof to the contrary the eyes of physicist.

If a triple of integers $a^n + b^n \equiv c^n$ exists, then it can map three nested integer edges hypercubes into each other (the centers of the nested hypercubes are aligned with the origin coordinates) while the volume of the small hypercube $a^n$ is equal to the difference between the volumes $c^n$ - $b^n$. Here the identity sign '$\equiv$' means independence from the scale and set partition of our construction, i.e. a triple of integers in meters, decimeters, centimeters, millimeters. It is easy to prove that the condition for the equality of volumes and the properties of the central symmetry, continuity of the formed constuction mutually exclude each other. To understand this let's mentally move the layer from set of points in space described by the formula $c^n$ - $b^n$ into a small cube $a^n$ and vice-versa.




**Picture 1. The figure of one layer (left) and set of layers in the octant (+, +, -)**

Here below a layer is defined as a set of points of a multidimensional spaces of real numbers $R^n$ between successively following hypercubes with integer edges $S_i = e_{i+1} \backslash e_i$ . The layer, like the whole n-dimensional figure, consists of elementary hypercubes $1^n$ in whole number space denoted as $Z^n$.

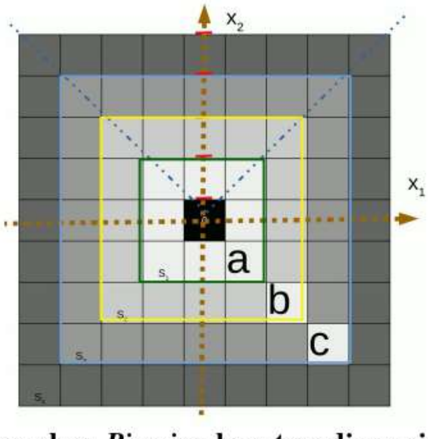
The designed construction of three nested hypercubes can be filled of layers step-by-step from the periphery to the center and vise-versa like building a frame house. This is the method used Euclid's *Elements* [2]. A layer from the c-*Large* hypercube must fit an integer number of times in the a-*Small* hypercube (due to the excess of large over small - two or more times), otherwise the central symmetry of the construction or the continuity of the ordered layers will be lost.

Here understanding the structure of the layer gives the following formulas:

$$S_i = (i+1)^n - i^n = \sum_{k=1}^{k=n-1} C_n^k i^k 1^{n-k} \qquad (2.1)$$

$$S_i = (i+1)^n - i^n = \bigcup_{k=1}^{k=n-1} C_n^k i^k 1^{n-k} \qquad (2.2)$$

The above formulas describe the layer as the difference of successive hypercubes having a common vertex coinciding with the origin. The first formula is derived from the Newton binomial. The second formula repeats the first one in terms of set theory. Another way of representing the construction under study, "the origin of coordinates located in the centers of hypercubes", is obtained by iterating n reflections from hyperplanes, i.e. multiplying $2^n$, which does not fundamentally change the kind of the formula. Both geometrical constructions are transformed into each other due to reflections from hyperplanes perpendicular to each of the n coordinate axes, or by cutting the figure and scaling.Each layer of hypercube have elements of dimensions n-1, n-2, ... 1 (hyper)faces and edges such elements is described by formula $i^k 1^{n-k}$ - i.e. *cuboid*. "At the destination" volumes of elements of each dimension must be identically equal the volume of the corresponding moved element, by virtue of the principle *incompressibility of the volume* of a solid body and the equivalence of the quantity elementary hypercubes $1^n$. These conditions lead to a system of n-1 equations that is not solvable for n > 2 not only in rational, but also in real numbers. To understand this we recall impossibility of constructing a right triangle, in which the hypotenuse is equal to the sum of the lengths of the legs. It is easy to verify that for these conditions, one of the legs will necessarily be equal to zero. Consequently, The construction of three nested hypercubes with integer edges is not exists in a space of whole numbers $Z^n$, n > 2 (*aporia* in terms of Ancient Greek philosophy), and there is no such triplet of numbers that would violate the Fermat's Last theorem.

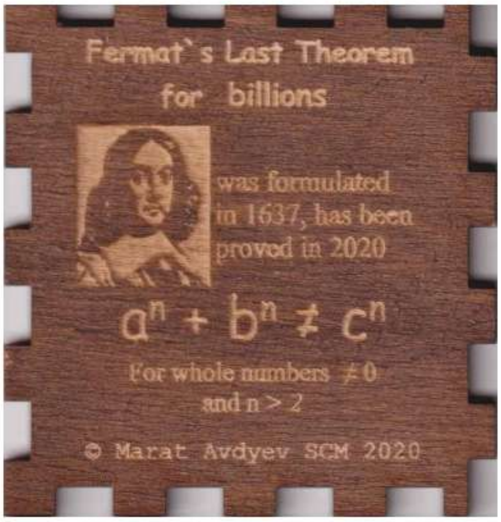
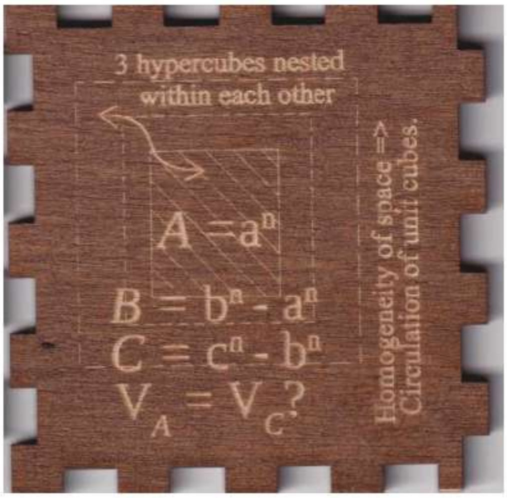
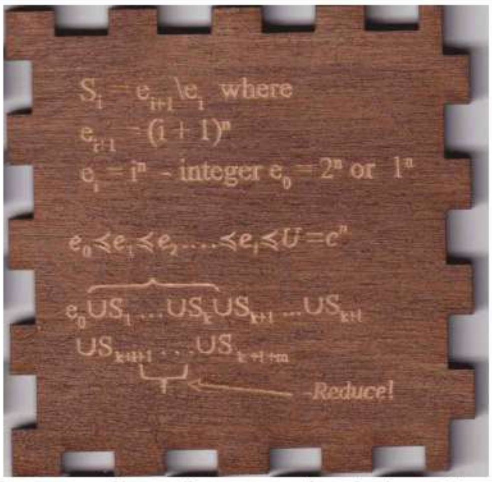



**Picture 2. Three nested hypercubes. *Piercing* by a two-dimensional plane. There is no parallax effect.**

(The thesis about the *piercing* (or penetrating) rather than cutting plane of a two-dimensional hypercube is easy to understand the basics of linear algebra **AX = B** (matrix form). It follows from the Kronecker-Capelli theorem that the set of solutions X to a system of linear equations forms a hyperplane of dimension n - rank **A** in $R^n$. For example, for a three-dimensional space and a two-dimensional intersection plane: dim (**X**) = 3 - 2 = 1. For 4 dimensional space and more dim(X) = 4 - 2 = 2 and so on. Therefore, a two-dimensional probe can be covered by a closed loop in a plane orthogonal to the piercing one, and it is appropriate to speak of *piercing* rather than *intersection*.)

In XVII century described physical approach was enough for proof, but not in XXI. More formal approaches is required [3].

*Tab 1 Scanning the faces of the 3D Cube*

| **Face of Cube** | **Comment** |
|---|---|
|  | Product name translated into Russian *The Fermat's Last Theorem for billions* referred to the registration of the database for computers, certificate number: RU 2020621077 Rospatent. Published 2020 Application number: 2020620372 registration: 11.03.2020 publication: 30.06.2020 "Proof of Fermat's theorem for Billions based on school knowledge". The theorem was formulated by Pierre de Fermat (pictured) in 1637 and proved by Marat Avdiyev in an original way in 2020. What follows is a reformulated statement of the theorem $a^n + b^n \neq c^n$ for integers and degree n > 2, the name and surname of the author of the short proof.<br>Proof of the opposite: |
|  | Let's compare the expression $a^n + b^n = c^n$ with the construction of three nested hypercubes having a common center at the origin with integer edges a, b, c. If the condition of equality of volumes in the discrete space of sets $A = \{a^n\}$, $C = \{c^n \setminus b^n\}$ and $V_A = V_C$, or cardinality $\|A\| = \|C\|$ take place, then elementary unit cubes $1^n$ can freely circulate between the layers of this symmetric construction, since the uniformity of space is postulated in physics. Here below it is easy to make sure that these two conditions mutually exclude each other in the space of integers denoted as $Z^n$ for n > 2.<br>It can be assumed sign '≡' in the expression above That means independence from the scale and set partition of our construction. |
| <br>The word Reduce! - [try] has been thrown as a challenge. Reduction is prohibited for n > 2. | The layer $S_i = e_{i+1} \setminus e_i$ is defined as the set of points in $R^n$, obtained by the operation of the difference of sets in the form of successive hypercubes with integer edges $e_{i+1} = (i+1)^n$ based on a series of natural numbers - an "empty box" with a thickness equal to 1 unit. The unit depend on partition (scale q). As a result, a chain of sets is formed — "empty boxes", nested. into each other. In the center of the whole structure is a hypercube of $1^n$ or $2^n$, depending on the parity of the partition (it does not matter). The chain of sets forms a large hypercube $c^n$, or universal set U. This formula does not allow for layer-by-layer reduction ($V_A = V_C$) for our centrally symmetrical construction of *homogeneous* material. Each layer $S_i$ is incommensurable with another $S_j$ in the $Z^n$, n >2.<br>=> Fermat's theorem is proved. |

**The set Theory and binary relations approaches**

It should be noted without change generality that the natural numbers in formula (1) are related as a < b < c, and the situation of equality of edges a = b is excluded due to the irrationality of $\sqrt[n]{2}$. The case of negative numbers can be considered by moving term into another part of the equation and substitution of variables - it is enough prove the theorem for the case of natural numbers a, b, c and generalize the result to whole numbers Z.

Let's consider inscribed hypercubes with edges, obtained from a series of consecutive natural numbers $N_1$, the centers which coincide with the origin of coordinates, and the faces are perpendicular to the axes coordinates Hypercubes $e_i$ with edges i based on a series natural numbers inscribed in each other form an increasing chain sets and the inclusion relation in the set U which is understood as large hypercube with edge c denoted as $I^n$:

$$e_0 \prec e_1 \ldots \prec e_k \ldots \prec e_{k+m} \ldots \prec e_{k+l+m} \prec I^n = U$$

$$e_0 \cup S_1 \cup S_2 \ldots \cup S_k \ \ldots \cup S_{k+l} \cup S_{k+l+m} \subseteq U \tag{3}$$

As mentioned above the *a-Small* n-cube $a^n$ is the set of layers indexed from 1 to k, the *b-Medium* $b^n$ is the set of layers from k+1 to k + *l* and the *c-Large* $c^n$ is the set of layers from k+*l*+1 to k + *l* + *m*. The layer is defined as the subset difference $S_i = e_i \setminus e_{i-1}$, i > 1. The first hypercube $e_0$ denote $1^n$ or $2^n$, in parity, but given the enclosures below, this detail is not leads to qualitative differences. A set partition one can see above. A chain of sets (3) define a topology in $I^n$. On the other hand, this formula describes a one-dimensional probe penetrating three nested hypercubes through a common center. The result of the *Cartesian Product* of two orthogonal probes can be seen in Picture 2 above, so the researcher can obtain a two-dimensional plane regardless of the space dimension. The topology of $I^n$ is induced on this subset because $0 \in Z^2 \subset R^n$. There is no parallax effect.
The mathematicians of ancient Greece introduced the concept of *incommensurability* of linear segments.

*Tab. 2. The postulates of Euclid in the Digital epoch*

| Figure in Euclidean space ($R^n$) provided central symmetry | Analogue in $Z^n$ set of hypercubes provided central symmetry | Dimension |
|---|---|---|
| A dot | $1^n$ | **0** and at the same time **n** depending on the situation |
| A linear segment | $i^1 1^{n-1}$ set of hypercubes cardinality = i lined up in a row or column one | 1 |
| A plane | $i^2 1^{n-2}$ set of hypercubes cardinality = $i^2$ ordered in a square | 2 |

The linear segments of length √2 and 1 are incommensurable. From these positions, each layer $S_i$ is incommensurable with another $S_j$ in the $Z^n$, n >2. It is easy to see that the analogous is true for sets of continuously following layers. The "uniqueness" of a layer can be formed by the condition: ∄ scale and set partition and natural i, j for which the *measure* $|S_i| = |S_j| \pm |S_{j-1}| + \ldots$ for n > 2, where |S| is *cardinality* i.e. quantity of $1^n$ in the investigated set. The *axiom of defining the measure* (volume in in terms of physics) over the set is violated. The measures of the set of layers S are not possess the *additivity* property in whole number multidimensional space $Z^n$ for n > 2. The operations of addition, subtraction, reduction, other comparison of different layers are being prohibited. So formula (3) describing structure of hypercube and understanding *measure axiom* for $S_i$ in $Z^n$ are enough for proof.

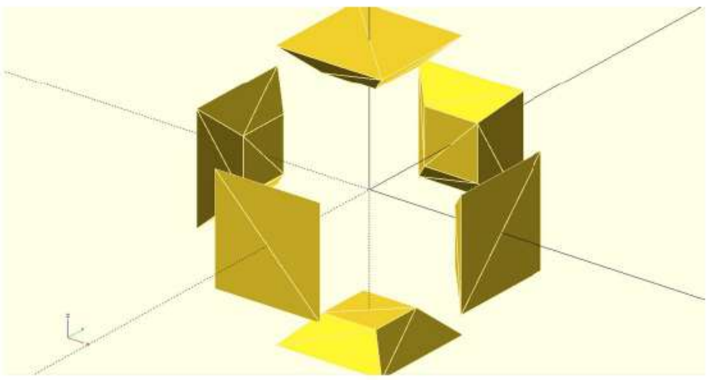
Let's define a continuous bijective function f U → U maintaining the fundamental properties of the our construction: central symmetry and continuous succession of layers. If ∃ f: *C* → *A* , where f is a bijective function mapping a subset *C* (*c-Large*) to a subset *A (a-Small)*, then this means $|C| = |A|$. Any function f is a superposition f = g ○ h, where g - bijective function within a layer h - bijective function between different layers. Let us focus on the *restriction* of the relation *g* to one specific layer $S_i$ . What is the product of $g|_{Si}$ ? By partition decreasing the thickness of the layers, it is possible to achieve a situation where a single layer $S_i$ from *C* is mapped to a set of layers {… $S_j$ ...}. According to definition of layer $S_i = e_{i+1} \backslash e_i$ $(i+1)^n - i^n$ and its structure $S_i = \sum_{k=0}^{k=n-1} C_n^k i^k 1^{n-k}$ each layer S = ∪ $d_k$ (where index k runs through values from 1 to n-1) *pairwise disjoint equivalence classes* of the elements $\mathbf{i^k 1^{n-k}}$ because of equivalence property the function *f* should transfer *pairwise disjoint equivalence classes* of the elements $\mathbf{i^k 1^{n-k}}$ (factor set) *separately*. Euclidean spaces of different dimensions are not homeomorphic: $R^n \Leftrightarrow R^k$ where n ≠ k is impossible [4].

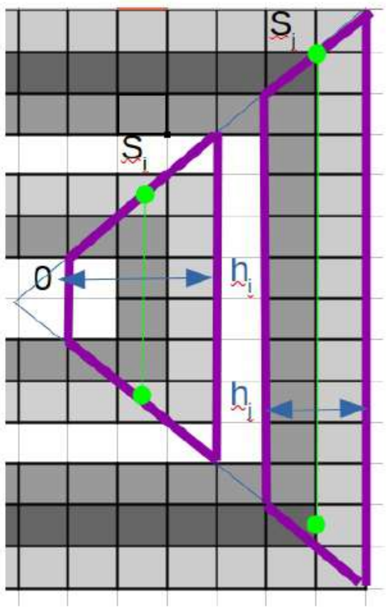
To ensure the simultaneous matching of the elements of the layer more than to one class is impossible due to the unsolvability for n > 2 of the stipulated below system of n-1 equations):

$j^{n-1} = i^{n-1} + (i-1)^{n-1}$ + . . . ( two or more terms) (3)

$j^{n-2} = i^{n-2} + (i-1)^{n-2}$ + . . . (two or more terms)

. . . this series of equations continues from n-1 to 1 power. (The observing construction has been filling of layers from the periphery to the center.) So ∄ equivalence function F in $Z^n$, n > 2 maintaining the fundamental properties of the our construction: central symmetry and continuous succession of layers.



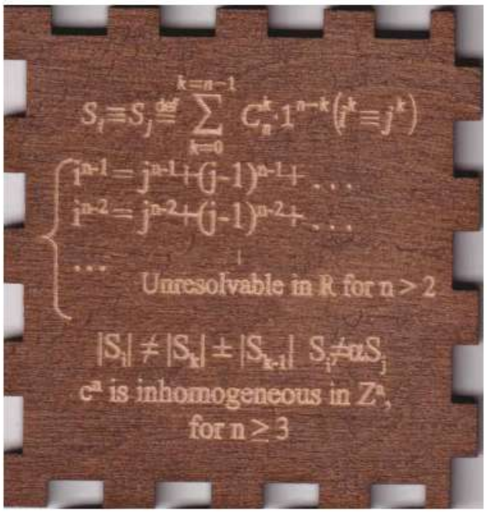


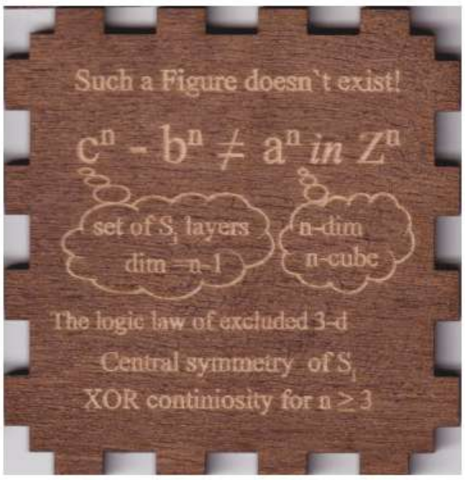

**Picture 3. ∄ equivalence function F in $Z^n$ , n > 2 maintaining the fundamental properties of the Construction: central symmetry and continuous succession of layers except two-dimensional case (trapezoid).**

For the special case $Z^2$ ∃ G, thanks to one equivalence class: comparison of trapezoid square is possible for ∀ i , j: ∃ $h_i$ and $h_j$ such as $S_i * h_i = S_j * h_j$ in Q numbers and by virtue of scaling for Z. In the middle of the 20th century french mathematician Claude Chabauty in 1938 defended his doctoral dissertation on number theory and algebraic geometry, actively applied the methods of symmetry of subspaces in analysis of Diophantine equations. Minhyong Kim a mathematician from the University of Oxford, researching hidden arithmetic symmetry of the Diophantine equations, said: “It should be possible to use ideas from physicists to solve problems in number theory, but we haven’t thought carefully enough about how to set up such a framework” [5]. The algorithmic unsolvability of Hilbert's Tenth Problem was proved by Yuri Vladimirovich Matiyasevech in 1970 at the St. Petersburg branch of the Mathematical Institute. V. A. Steklov RAS [6]. From a philosophical standpoint, formula (1) has a contradiction between form (central symmetry) and content (volume) for n >2.

*Tab 3 Scanning the faces of the 3D Cube (continues)*

<table>
<tr><th>Face of Cube</th><th>Comment</th></tr>
<tr><td>


</td><td>Why does Pythagorean triples exists specifically for the two-dimensional case, i.e. $a^n + b^n = c^n$ for integers and degree n = 2? Based on the central symmetry of the construction of three nested hypercubes, we consider only one axis. Rays drawn from the origin to the vertices of one face dissect the hypercube into 2n regular hyperpyramids. In the particular two—dimensional case - on triangles. Any arbitrary layers are commensurate, as well as sets of layers $\{\ldots S_i \ldots\}$ and $\{\ldots S_j \ldots\}$ are trapezoids of height $h_1$ and $h_2$. - by the number of layers in the set. For arbitrary averages by the line of the trapezoid $S_i$ , $S_j$, you can choose the corresponding. the number of layers and make the volumes $V_A = V_C$. equal with respect to symmetry and homogeneity of space.</td></tr>
<tr><td>


</td><td>Definition of the layer as a set of points in $R^n$ obtained by the operation of the difference of sets $S_i = e_{i+1} \backslash e_i$ or algebraic expression $(i+1)^n$ - $i^n$ via the Newton's binomial theorem: $S_i = \sum_{k=0}^{k=n-1} C_n^k i^k 1^{n-k}$ here the coefficients of $C^k_n$ are the same for any layer *i*. Therefore, an identical comparison of the volumes (capacities of sets) $S_i$ and $S_j$, regardless of the partition and scale *q* (see above), means an element-by-element comparison of each dimension *k* separately (equivalence class). This leads to an unsolvable system of equations even in real numbers R, not to mention integers, for n > 2. Incommensurability of layers means heterogeneity of space. This conclusion contradict to physics and axiom of measure in math. As a result, a logical contradiction was revealed.</td></tr>
<tr><td>


</td><td>From the point of view of physics, we compare hypercubes with integer edges: $a^n = c^n — b^n$. On the left is a homogeneous, isotropic, symmetrical figure of dimension n, and on the right is a set of layers of dimension n—1 that is either asymmetric or inhomogeneous, depending on the methods of construction and partitioning (scale). From the logical principle of the exclusion of the third follows: a centrally symmetric construction of three nested (hyper)cubes with integer edges a, b, c doesn't exist in reality — there is *aporia*. => The theorem is proved.</td></tr>
</table>

## Conclusion

In the XVII century, there was still no separation between physics and mathematics in science, and it can be assumed that Pierre de Fermat used an interdisciplinary approach. In modern physical cosmology, the fundamental principle is the idea that the spatial distribution of matter in the Universe is *homogeneous* and *isotropic* when viewed on a sufficiently large scale, as a result of the evolution of matter, laid down by the Big Bang. The assumption of free circulation of hypercubes from $V_A \leftrightarrow V_C$ and vice versa corresponds to the physical phenomenon of diffusion.
Since Pierre de Fermat claimed that "he found truly wonderful evidence, but the fields are too narrow to fit it." Solving cumbersome equations is the wrong way to find evidence. From these positions, Fermat's Great Theorem is proved by careful consideration with just one glance, as in the ancient Indian treatises on mathematics, where the proof in one drawing was accompanied by only one word: *Look*! Perhaps through insight the attentive reader will be able to see the layers, the lack of additive property of equality of their volumes, the inevitable violation of the symmetry of the figure during the circulation of hypercubes.

Photo 4 above of author's 3D wooden hypercube with the proof Fermat's Last Theorem.
Edge of the cube -72 mm, thickness of plywood -4 mm

## In a few words

If a triple of integers $a^n + b^n \equiv c^n$ exists, then it can map three nested integer edges hypercubes into each other (the centers of the nested hypercubes are aligned with the origin coordinates) while the volume (cardinality of the set) of the small hypercube $|a^n|$ is equal to the difference between the volumes $|c^n \setminus b^n|$. Because of equivalence of volumes (*measures*) there should exist continuous bijection function *f :* $\{c^n \setminus b^n\} \rightarrow \{a^n\}$ so single layer from the set $\{c^n \setminus b^n\}$ is mapped to a connected set is of layers into $|a^n|$. But $\nexists$ such function in $Z^n$, $n > 2$ maintaining the fundamental properties of the construction: *central symmetry* and *continuous succession* of layers based on a series of natural numbers $N_1$. The construction of three nested hypercubes with integer edges is not exists in a space of whole numbers $Z^n$, $n > 2$ (*aporia*).

It turned out that Pierre de Fermat's statement is not a figure of speech, that it should be taken literally. The mathematician did not lie at all when talking about the possibility of recording the main ideas of the proof in the fields of Diophantus arithmetic. At least six faces of the wooden cube were enough. From a philosophical standpoint, Fermat's Last theorem contains an irremediable conflict between form and content.

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